Due Fri

2.2 – Evaluating Determinants by Row Reduction

Theorem 2.2.1 Let A be a square matrix. If A has a row of zeros or a column of zeros, then $\det(A) = 0$.

Theorem 2.2.2 Let A be a square matrix. Then $\det(A) = \det(A^T)$.

Theorem 2.2.3 Elementary Row Operations

Let A be an $n \times n$ matrix.

a) If B is the matrix that results when a single row or single column of A is multiplied by a scalar k, then $\det(B) = k\det(A)$.

b) If B is the matrix that results when two rows or two columns of A are interchanged, then $\det(B) = -\det(A)$.

c) If B is the matrix that results when a multiple of one row of A is added to another or when a multiple of one column is added to another, then $\det(B) = \det(A)$.

pf (a): Let B be the matrix that results from multiplying the i^{th} row of A by k.

$\det(A) = \sum_{l=1}^{n} a_{il}C_{il}$ by expanding along the i^{th} row

$$\det(B) = \sum_{l=1}^{n} b_{il}C_{il} = \sum_{l=1}^{n} ka_{il}C_{il} = k\sum_{l=1}^{n} a_{il}C_{il}$$

$$= k\det(A) \checkmark$$

$$\begin{vmatrix} 2 & 1 & 3 \\ 4 & 6 & 8 \\ 5 & 7 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 & 3 \\ 2 & 3 & 4 \\ 5 & 7 & 2 \end{vmatrix}$$

Theorem 2.2.4 Let E be an $n \times n$ elementary matrix.

a) If E results from multiplying a single row of I_n by a nonzero number k, then $\det(E) = k$.

b) If E results from interchanging two rows of I_n, then $\det(E) = -1$.

c) If E results from adding a multiple of one row of I_n to another, then $\det(E) = 1$.

Special case of Thm 2.3.3

12. Evaluate the determinant of the matrix by first reducing the matrix to row echelon form and then using some combination of row operations and cofactor expansion.

$$\begin{bmatrix} 1 & -3 & 0 \\ -2 & 4 & 1 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 1 & -3 & 0 \\ -2 & 4 & 1 \\ 5 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 0 \\ 0 & -2 & 1 \\ 0 & \boxed{13} & 2 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & \frac{17}{2} \end{vmatrix}$$

$R_2 \to R_2 + 2R_1$

$$\begin{array}{ccc} -2 & 4 & 1 \\ 2 & -6 & 0 \\ \hline 0 & -2 & 1 \end{array}$$

$R_3 \to R_3 - 5R_1$

$$\begin{array}{ccc} 5 & -2 & 2 \\ -5 & 15 & 0 \\ \hline 0 & 13 & 2 \end{array}$$

$R_3 \to R_3 + \frac{13}{2}$

$$\begin{array}{ccc} 0 & 13 & 2 \\ 0 & -13 & \frac{13}{2} \\ \hline 0 & 0 & \frac{17}{2} \end{array}$$

$$\begin{vmatrix} 1 & -3 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & \frac{17}{2} \end{vmatrix} = (-2)\left(\frac{17}{2}\right) \begin{vmatrix} 1 & -3 & 0 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{vmatrix} = -17$$

-17

Q: Is this true:

$$\begin{bmatrix} 1 & -3 & 0 \\ -2 & 4 & 1 \\ 5 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 0 \\ 0 & -2 & 1 \\ 0 & 13 & 2 \end{bmatrix}$$

NO! :(

In #16 and 20, evaluate the determinant, given that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6.$$

16. $\begin{vmatrix} g & h & i \\ d & e & f \\ a & b & c \end{vmatrix} = 6$

20. $\begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g+3a & h+3b & i+3c \end{vmatrix} = -12$

$R_2 \Rightarrow 2R_2$ doubles the determinant

$R_3 \Rightarrow R_3 + 3R_1$ leaves the determinant unchanged.

Theorem 2.2.5 If A is a square matrix with two proportional rows or two proportional columns, then $\det(A) = 0$.

We can obtain a row of zeros with one row operation.

Consider $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and $3A = \begin{bmatrix} 6 & 9 \\ 12 & 15 \end{bmatrix}$

$\det(A) = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = -2$ $\qquad$ $\det(3A) = -18$

$3\det(A) = -6$

$3 \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} 6 & 9 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 12 & 15 \end{vmatrix}$